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DEPARTMENT OF EDUCATION

CURRICULUM GUIDE

For

GRADE EIGHT MATHEMATICS

September, 1966

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TEXTS AND REFERENCES

Effective September, 1966 either of the two textbooks below may be used in Grade VIII classrooms.

1. *Seeing Through Mathematics, Book 2,* Special Edition
   Van Engen et al (W. J. Gage and Co.)
2. *Exploring Modern Mathematics, Book 2* (Canadian Edition) Keedy et al.
   (Holt, Rinehart and Winston of Canada, Ltd.)

SUPPLEMENTARY REFERENCES

1. *Teaching Guide Seeing Through Mathematics, Book 2,*
2. *Response Verification Book Seeing Through Mathematics, Book 2,*
   Special Edition
3. *Exploring Modern Mathematics, Book 2,* Teacher's Edition
   (Canadian)
4. *Charting the Course for Arithmetic,* Hartung et al.
   (Scott, Foresman and Company)

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INTRODUCTION

A. BACKGROUND

It is desirable that the Grade VIII mathematics teacher has taught the new Grade VII program. Failing this, it is essential that the Grade VIII teacher carefully study the content of the Grade VII text used by the class. It is also desirable that the teacher spend some time becoming aware of the content of the Grade V and VI programs. The effort required to gain such background knowledge will pay rich dividends in terms of more effective instruction and increased teacher confidence.

Several features of the present Grade VII programs make a detailed knowledge of the subject matter more important than was the case with the previous authorization.

1. Definitions — Definitions which differ slightly from traditional usage in mathematics have been introduced and are rigorously followed in the texts. The teacher who operates from different definitions than her students will cause confusion.
2. Symbolism — Many of the symbols introduced in Grade VII did not appear in traditional texts. Other symbols are used with a different or more precise meaning than previously. Teachers who have not taught from the Grade VII text of the series will be confronted with the task of adapting to a symbolism with which the pupils will already be familiar.
3. Sequential Nature of Materials — Both Grade VIII authorizations place heavy emphasis upon the sequential development of concepts. Many ideas introduced at the Grade VII level give the pupil a familiarity with the concepts involved. The Grade VIII text assumes this background knowledge as it develops the idea further. To do the best possible job of teaching it is essential that the teacher be aware of the stage of development of a particular topic.

B. EMPHASIS ON UNDERSTANDING

Both Grade VIII authorizations present materials in a manner designed to develop in pupils an understanding and appreciation of the structure of mathematics. Opportunities are provided to stress the common characteristics of various number systems, and to show the repeated applicability of certain principles and properties. It is the responsibility of the teacher to present the materials and to direct the discussion in such a manner that the greatest possible number of pupils will grasp these generalizations. The teacher who understands and appreciates the beauty of mathematical patterns can do much to help pupils find enjoyment in their mathematical studies.

C. NEED FOR CONTINUOUS DRILL

The present texts place increased emphasis on understanding. Repeated drill and review must follow this understanding if concepts are to be established firmly. No mathematics text yet devised can provide the best amount of drill for all pupils. Teachers must supplement the text to meet the needs of their particular pupils. Because there is a tendency on the part of teachers using a text for the first time to move slowly with the unfamiliar portions and to cover traditional topics hastily, such teachers must be very careful that they incorporate into their instruction provision for sufficient practice to maintain and develop computational skills.

Elsewhere in this guide the teacher will find a suggested time allotment for various units. If the teacher is having difficulty keeping on schedule he should note that possible omissions are listed for each text. These topics rather than review or drill should be omitted when it is necessary to gain time.

D. TEACHER PREPARATION

It is impossible to use a mathematics text to best advantage without being thoroughly familiar with its content. To acquire this familiarity with the new texts will require many more hours of teacher time than was the case with a traditional text. Only when the teacher is aware of the concepts and sequence suggested for the development can decisions for a particular class be made effectively.

Although the text generally follows one pattern of lesson presentation, the alert teacher will use many variations. No one method can consistently maintain interest. Some lessons can be effectively presented by following closely the pattern of questions outlined in the text. Other topics may best be presented directly by the teacher. In still other lessons the pupils might advance most successfully by reading the text for themselves. To make the best judgements about teaching procedures for a given topic and a given class, the teacher must spend considerable time in preparing each lesson.

E. VISUAL AND MANIPULATIVE MATERIAL

The textbook is undoubtedly the most important aid in planning the school mathematics program. However, to be interesting and meaningful to students, the program must be supplemented by a careful selection of visual and manipulative aids. These materials should be easily accessible and should be used systematically to help clarify and simplify the mathematical ideas treated in the textbook.

It is impossible to provide anything like a complete list of visual and manipulative aids. Generally, however, such materials can be divided into the two categories:

1. Commercially prepared material
2. Raw material

Commercial materials designed to exhibit some of the properties of the Hindu-Arabic numeration system, such as Cuisenaire rods or Stern blocks, can be used to illustrate many structural characteristics of a mathematical system. Prepared geometric models of polyhedra enable the student to identify many important properties of solids. Mathematical games and puzzles may help the student to visualize certain spatial relationships more clearly. Such materials are regularly advertised for sale in professional journals. However, since commercial materials are usually prepared for a specific purpose their usefulness is limited. Moreover, their structure is complete and if students are limited to these kinds of materials they miss the opportunity of building structures for themselves.

Priority should be given in the mathematics classroom to the provision of raw materials which will permit the student to produce his own models and visual aids. For example, in the study of polyhedra the student can construct models with wooden rods of various lengths, and modelling clay. It is likely that he will become more familiar with the structure of polyhedra in general than the student who is only provided with ready-made plastic models to handle.

Mathematics classrooms should be equipped with kraft paper, string, peg board, modelling clay, pins, wooden or plastic rods, and other materials which can be used to help students discover mathematical relationships on their own. To get maximum use from such materials there should be regularly scheduled laboratory periods in which students are free to experiment and explore. The main purpose of such periods should not be to produce models for display. The object should be to encourage students to become involved in the discovery and application of mathematical principles and relationships through their own manipulations.

EXPLORING MODERN MATHEMATICS
Book 2

A. General Considerations

1. Teacher's Edition

Exploring Modern Mathematics, Book 2, Teacher's Edition is available from Holt, Rinehart and Winston of Canada, Ltd., as a separate publication. Each teacher of this program is strongly advised to have a copy of the *Teacher's Edition*. This book contains answers to each of the exploratory exercises and practice exercises, and presents an explanation of the approach and philosophy adopted in the program. There are valuable suggestions made on how to use the exploratory exercises, and how to maintain skills. A useful overview of each chapter is provided as well as lesson-by-lesson hints and suggestions.

Exploring Modern Mathematics, Book 1, Teacher's Edition will also be necessary for the teacher for the following purposes:

a. as the basis of the approach to the content of the Grade Seven program,

b. as background information for the teacher new to the modern mathematics program, and

c. as information for the student entering this program with a different mathematics background.

2. Semi-Programmed Nature

By using the Let's Explore exercises, each student is encouraged to discover for himself each new concept. THESE EXPLORATORY EXERCISES ARE NOT TO BE GRADED. It is considered essential that the teacher using the text, *Exploring Modern Mathematics, Book 2,* work through the Let's Explore exercises in advance of the class period in order to anticipate problems which the student may encounter.

It is also the teacher's responsibility to determine whether certain sections in the exploratory exercises may be deleted for superior students. In some instances, a short discussion may replace such sections. Teachers who have taught this program have found that in general the exploratory exercises make a useful, stimulating and vital contribution to their teaching. Certain exercises may require supplementary questions for students who are having difficulty.

Experience has shown that it is useful to spend a short time at the beginning of the class period to make the student aware of a gap in his knowledge and to create some curiosity about the exploratory exercise to follow. The class should then proceed, on their own, to work through the exercise, the teacher being careful to give assistance only to those students who obviously are making no progress. Students who finish first might be encouraged to do appropriate questions in the practice exercise which usually follows. It is important to bring the class together following the exploratory exercise to discuss the concepts under consideration. On some occasions, it will be necessary for the teacher to work at the blackboard with the entire class, developing the concepts again and reviewing the conclusions.

The assignment of a Let's Explore exercise as homework has not been found satisfactory, since experience indicates that individual differences within classes are often great, and the teacher must be available to provide assistance. Teachers who have taught this program have found that most students can do the exploratory exercises in class, and that sufficient understanding can be gained for students to begin the practice exercises which can then be finished as homework.

3. Programmed Supplement

The student who has been absent for some time or who is having difficulty understanding the concepts may find the textbook unsatisfactory because this text was not designed to allow a student to work completely on his own. The *Programmed Supplement, Exploring Modern Mathematics, Book 2,* contains the Let's Explore exercises with answers developed in more de-

tail, using a workbook-type format. Teachers may find that having several copies of this publication for students' use will be necessary to avoid undue repetition of lesson content previously taught.

The student who begins the program, *Exploring Modern Mathematics, Book 2,* without having used the *Book 1* program will find that the *Programmed Supplement, Exploring Modern Mathematics, Book 1,* will be an efficient aid to gaining the mathematical concepts needed as background for understanding the second level of the program.

4. Schedule of Units and Minimum Program

a. The following approximate time schedule appears to be suitable for most students:

	Minimum		*Maximum*	
Chapter 1 — The System of Integers	5	-	6	weeks
Chapter 2 — Geometry: Congruence, Construction and Circles	4	-	5	weeks
Chapter 3 — Exponents and Scientific Notation	5	-	6	weeks
Chapter 4 — The System of Rational Numbers	3	-	4	weeks
Chapter 5 — Geometry: Areas, Regular Polygons and Circle Graphs	3	-	4	weeks
Chapter 6 — More About Integers and Rational Numbers	3	-	4	weeks
Chapter 7 — Solving Equations	4	-	5	weeks
Chapter 8 — Polynomials	4	-	5	weeks
TOTAL	31	-	39	weeks

b. Content which may be omitted by classes or individual students of moderate ability is listed below. Topics considered optional by the authors are marked by an asterisk (*) or indicated as optional in the title of the section in the textbook. Sections so indicated are not part of the minimum program. Note that Chapter 9 is not included in the Grade Eight minimum program. Chapter 1 of the Grade Nine program includes the content of Chapter 9 of the Grade Eight textbook.

In Chapter 1, *The Systems of Integers,* omit:

Properties of Addition, including:
pp. 20-21 Section 1.8, Let's Explore
pp. 21-22 Section 1.9, Let's Explore
p. 22 Exercises, numbers 1, 2, and 3

In Chapter 3, *Exponents and Scientific Notation,* omit:

p. 137 Exercises, number 3
p. 140 Section 3.11, Let's Explore
p. 144 Section 3.14, Let's Explore, number 8
p. 150 Exercises, number 6
p. 152 Exercises, number 4

In Chapter 4, *The System of Rational Numbers,* omit:

p. 171 Section 4.2, Let's Explore
p. 172 Section 4.3, Let's Explore
pp. 175-176 Section 4.4, Let's Explore
p. 189 Section 4.13, Let's Explore

In Chapter 6, *More About Integers and Rational Numbers,* omit:

p. 256 Exercises, number 3 (at top of page).

5. Sequence of the Program

Only after considerable experience with the text and after very careful thought, should the teacher contemplate changing the sequence of the course. It is further suggested that teachers do not omit any section other than those marked "(*)", "Optional" or those suggested in this curriculum guide.

Teachers presenting this course for the first time may find it useful to omit all sections marked "Optional" for average and below average classes, in order to complete the program.

6. Proofs and Definitions

Proofs and definitions are more rigorous at the Grade VIII level than at the Grade VII level. The teacher must encourage an awareness of the nature of proof. However, experience indicates that some of the proofs in the Grade VIII program are too difficult for a number of students. It is likely that the teacher will find it necessary to do such proofs for his students.

7. Review Exercises

Teachers are referred to *Exploring Modern Mathematics, Book 2, Teacher's Edition,* pp. XV-XVI, for suggestions about the use of review material.

8. Tests

A separate publication entitled *Tests for Exploring Modern Mathematics, Book 2,* is available from Holt, Rinehart and Winston of Canada, Ltd. Teachers will find this publication an aid to securing an accurate evaluation of the students' progress.

9. Further comments

Certain information contained in the *Teacher's Edition, Exploring Modern Mathematics, Book 2,* is essential to the proper understanding of the program. The integration of arithmetic, algebra and geometry where feasible; the increased attention to reading; the emphasis on mathematical structure; the development of proofs; and the cautions about forced verbalization of concepts should be noted by the teacher. Pages XI and XII of the *Teacher's Edition* contain paragraphs useful in understanding the nature of this series of textbooks.

B. Lesson Comments

CHAPTER 1: THE SYSTEM OF INTEGERS — 5 - 6 WEEKS

Optional Content

Properties of Addition

pp. 20-21 Section 1.8, Let's Explore
pp. 21-22 Section 1.9, Let's Explore
p. 22 Exercises, number 3

The construction of the negative integers is based on abstract principles and so differs from the traditional approach with its geometric or vector interpretation. A recall of the concepts in *Book 1* will review the properties of the number systems already studied in Grade Seven. A new number system, to be named the integers, is developed to obtain additive inverses for all the numbers so that there will be closure under subtraction.

The concept of absolute value is introduced and a definition stated. This concept is then utilized in developing definitions of operations with integers. Following the definition of addition and subtraction algebraically, the number line is presented as a geometric interpretation. There are problems which give practical application of the integers to the physical world.

Other important sections in this chapter deal with number sentences and number phrases which lead to the simplification and solution of equations. Conditional sentences and their converses introduce mathematical reasoning, a concept which will be important in later work

with proofs. Also, integers are used as an introduction to graphs on a plane to picture the solution set of a number sentence.

Mathematical Systems, pp. 1-6

Before the teacher proceeds with the development of the system of integers, essential background concepts should be established with the class. Mathematical systems studied in the Grade Seven program, *Exploring Modern Mathematics, Book 1,* were:

1. natural numbers, Chapter 1
2. whole numbers, Chapter 1
3. clock arithmetic, Chapter 5
4. modular arithmetic, Chapter 5
5. number system of arithmetic, Chapter 6

It is suggested that the teacher review the essentials of a mathematical system, including the concepts of:

1. a set
2. one or more operations
3. properties of the operations
4. special members of the set.

The content included in Chapter 5, on pages 196 through 225, *Exploring Modern Mathematics, Book 1,* and particularly the topics including additive inverses, is essential to an understanding of the concepts introduced in the first chapter of *Book 2.*

The teacher should not overemphasize the arithmetic of the five-minute clock, since it is possible that students will believe that the results of the operations in this system must be memorized. Perhaps a six-minute clock could also be used in order to place the emphasis on the general characteristics of mathematical systems, and not on any particular system.

Integers, pp. 6-11

The review at the beginning of Chapter 1 leads to the construction of a new number system, the integers, which is to be closed under subtraction. The approach in which a number system is invented will be a new experience to the students. The set will consist of zero, all the natural numbers and an additive inverse for each natural number 'n'. An arbitrary method of symbolizing additive inverses is agreed upon. It is important to stress that the correct meaning of '$^-$n' is 'additive inverse of n' or simply 'inverse n'. Later this may be called 'negative n'.

The number line, page 7, is used to illustrate the three sets into which integers are separated and to place them in order. Absolute value, page 10, is another important concept arrived at by mathematical agreement. Mastery of the definition and application of absolute value is required because of its importance in definitions of addition and subtraction with integers.

Addition for Integers, pp. 11-23

This will be the students' first experience with mathematical assumptions from which conclusions may be derived. It may prove to be a difficult section. After the invention of the integer, it is desirable to define the operation of addition in this set. It is assumed that addition can be defined in such a way that it will be commutative and associative. This definition is developed in several steps:

1. Addition is already defined for the addition of two whole numbers and for an integer and its additive inverse;
2. By agreement zero is defined as the additive identity. Thus, $0 + {}^-m = {}^-m + 0 = {}^-m$.
3. It is decided that if the commutative and associative properties are to hold, a definition for adding negative integers is to be in the form given in the display on page 15.
4. The next step is to define the sum of a positive and a negative integer by a similar procedure.
5. The complete definition for addition of integers is summarized in the display on page 18.

Properties of Addition, p. 20

This is an optional section. The properties of addition have been assumed thus far and it is suggested that the proof not be included in the course content unless there is very good reason to assume that students are either capable of benefiting or are sufficiently interested. However, pupils should be aware that it is possible to prove that the definition of addition that has been accepted ensures that addition is commutative and associative.

Subtraction, pp. 23-24

Subtraction of integers is defined in terms of additon and is based on the concept of subtraction in clock arithmetic introduced in *Book 1*.

Addition and Subtraction on the Number Line, pp. 25-28

This section may be considered optional if the section "Properties of Addition" on page 20 was understood. By using the number line the commutative and associative properties may be established intuitively although this does not constitute conclusive mathematical proof.

Thermometers, p. 28 and Other uses of the System of Integers, pp. 29-31

Thermometers, card games, profit and loss, and stock markets provide practical application problems to illustrate the usefulness of the system of integers in the physical world.

Number Sentences, pp. 31-34, and Number Phrases, pp. 34-40

An equation is defined in *Book 1* as any sentence with "=" for its verb (p. 52). *Book 2*, (p. 32), introduces the concept of inequality as a sentence which may have one of six symbols for a verb. Each of these may be negated by changing the verb to "is not ——.", (p. 32).

The next step in the development of the solution of equations is a consideration of number phrases. Such phrases are either the name of some number or can name a number for some replacement of the variable. Students are led to the definition that number phrases are considered equivalent if they name the same number for all numbers in the replacement set. This concept is then put into use for simplifying phrases and finally for solving equations. These sections will give some difficulty and require considerable teacher guidance. Proficiency is not expected at this stage. An understanding of the concepts involved will provide a basis for sections occurring later in the text.

Conditional Sentences, pp. 40-45

The concepts involving conditional sentences and converses should be considered introductory content. It is not considered necessary to pursue these ideas in depth at this stage of the program.

Graphs on a Plane, pp. 45-55

The introduction to graphs on a plane does not appear difficult for most students. Note that the dots representing integers, as illustrated on page 46, should not be connected with lines.

CHAPTER 2: **GEOMETRY: CONGRUENCE, CONSTRUCTIONS AND CIRCLES**

4 - 5 WEEKS

This chapter may prove to be one of the most interesting and least difficult sections of the course. It provides many exercises for pupil activity.

Chapters 3 and 7 of *Book 1* introduce geometrical concepts at the Grade Seven level. A short review may be necessary once there is considerable review incorporated into this chapter.

So that the students will have an adequate geometrical vocabulary for future use, it may be necessary to give considerable importance to the meaning of definitions included in this chapter. As a preliminary to more formal geometric reasoning and proofs, informal proof is introduced in many exercises. Formal proof in geometry is left for *Book 3*.

Geometric construction gives practice in following directions carefully. It is not necessary that students be required to state a formal description of the process of construction of geometric figures but they should be able to reproduce all required figures correctly.

Geometric Figures, p. 59

If the teacher is not familiar with *Book 1* of this series, Chapter 3 of the *Teacher's Edition* should be consulted since considerable guidance is given for the "point-set approach" to geometry. The brief review, pages 59 to 61, may be adequate but it might be expanded to make certain that the students have remembered set notation and the definitions of segment, ray, open segment, line, half line, angle, measure, figures on a plane, etc.

Congruence, p. 61

Congruence is a new concept but does not generally present any difficulty when considering closed curves. Care must be taken to be precise in meaning when measure, equality and congruence are being considered simultaneously as in question 5, page 64. Figures are congruent if they will "fit together" exactly. Angles must have the same measure to be congruent and segments must have the same measure to be congruent. If two figures are equal then these are not two different figures but rather two names for the same figure.

Compass and Straight Edge, p. 65

The construction exercises pages 65 to 81 (to copy an angle, to bisect an angle, to bisect a segment, to draw a perpendicular to a line) should prove interesting to the students and present little difficulty to those who are capable of reading and following instructions. Less keen students will require more supervision. No formal proofs for the construction are required. It is suggested that students will be convinced intuitively that the constructions are sound.

Parallel Lines, p. 81

Construction, definitions and informal proof, pages 81-91, may prove to be a difficult section requiring careful teacher guidance. The vocabulary relevant to parallel lines and transversals as summarized on pages 84 and 85 should be mastered.

Pages 85 to 90 introduce geometric proof of the congruence of angles formed by a transversal and parallel lines. These are informal proofs developed through Let's Explore exercises but teacher guidance to establish correct mathematical procedure in preparation for later work requiring written proofs may be necessary. The importance of conditional sentences and their converses may require reviewing, (see pages 40 to 45). Compare displays on pages 89 and 90 with the display on page 88 to note examples where the converses of conditionals are also true. Most students will accept and utilize congruence of angles formed by transversals but many will not remember the procedure for proof.

Note that the description of the construction of parallel lines on pages 90 and 91 is an introduction to formal geometric construction.

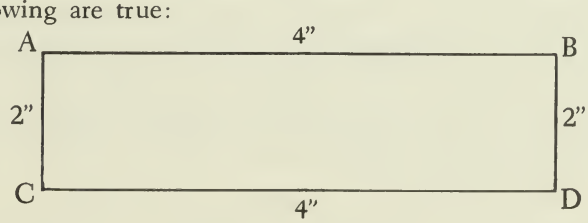
Quadrilaterals, p. 91

Pages 91 to 95 may be given as reading exercises but additional discussion and drill will be necessary to clarify the relationships among the quadrilaterals brought to the student's attention by the Vern diagram on page 95 and the exercises on pages 96 and 97. Several of the questions on these pages may offer difficulty but they are important enough to warrant attention.

It should be stressed that angles and sides are considered *congruent* to each other but are *equal* in measure, e.g. $m \angle ABC = 60°$ and $m\ \overline{KL} = m\ \overline{CD}$. Note that $\overline{KL}$ is used as a shorter form of $m\ \overline{KL}$.

For the figure below, the following are true:

$\overline{DA} = \overline{AD}$
$\overline{DA} \cong \overline{BC}$
$\overline{DA} \neq \overline{BC}$
$m\ \overline{DA} = m\ \overline{BC}$



Circles, p. 98

Pages 98 to 104 introduce the traditional concepts of the circle by exploration, discovery and definitions. Precise language is stressed. The circle is a set of points forming a plane figure. The distance around the circle is the circumference and is, therefore, a number. The diameter and radius are segments and, therefore, not distances. However, in circle formulas, the letter "d" represents the length of the diameter and the letter "r" represents the length of the radius. Note the introduction on page 103 of a new symbol ($\approx$) meaning "is approximately equal to". Note: Because of the geometric nature of this chapter there is little practice in arithmetic skills. It might be advisable to assign additional exercises similar to those on page 81. For those classes whose teachers are concerned because of the lack of problem-type questions, additional exercises may be assigned.

CHAPTER 3: **EXPONENTS AND SCIENTIFIC NOTATION** 5 - 6 WEEKS

Optional Content

p. 137 Exercises, number 3
p. 140 Section 3.11, Let's Explore
p. 144 Section 3.14, Let's Explore, number 8
p. 150 Exercises, number 6
p. 152 Exercises, number 4

Multiplication and division of powers with like bases are developed without difficulty until negative integers as exponents are encountered. A degree of mathematical sophistication is required to understand the process by which rules are developed for multiplication and division when one or both exponents are negative. A brief section deals with expanded numerals having negative exponents. The section on a power leads into a definition of multiplication of integers, a concept which was not discussed when integers were considered in Chapter I. Scientific notation is introduced following exercises dealing with multiplication and division by powers of ten. This notation is applied to the metric system of measure and to approximations.

Powers of Numbers, p. 113

Expanded numerals were first introduced in *Book I*, page 9, and it is to be expected that this topic will require little review.

Multiplying Powers, pages 114 to 116, as an exploratory discovery should not present any difficulty with the exception of additonal discussion of the language used in the fact display, page 116. Similarly, the discovery of dividing, using exponential notation, pages 118 to 120, develops naturally with little difficulty except for the interpretation of the display fact, page 119.

The Review Practice, pages 116 and 117, is valuable as a review of calculations using integers and numbers of arithmetic. (See Chapter 6, *Book I*, for the latter number system.)

Negative Integers as Exponents, p. 120

This is a difficult section. It requires the ability to follow through the mathematical procedure of supposing or making an assumption about the use of negative integers as exponents. The steps by which these concepts are developed are:

1. Page 120, Exercise 1. Division of powers with like bases by factorization and simplification.
2. Page 121, Exercise 2. With the same group of examples as in Exercise 1, the results are obtained when the rule on page 119 is applied as if it also held for $x < y$.
3. Comparing the simplified forms in Exercises 1 and 2 it is decided how powers with negative exponents should be defined if this rule for division is to hold. This definition is given in the display, page 122.
4. Page 122 gives practice in applying this definition.
5. Page 123 leads to the conclusion stated in the display page 124. This extends the definition so that the integer in the exponent may be positive, negative or zero.

6. Page 124 contains a short exercise of application.
7. Page 125 has several exploratory exercises to ascertain the validity of the converse of our procedure, i.e., if we make our definition as we did, then does the rule on page 116 apply for any integral exponent?
8. This is followed by a set of exploratory examples on pages 127 to 128 to show that the rule for division of powers also works for any integral exponents.

It is quite possible that many students will be capable of following each individual exercise with understanding but that they will become lost in the long view of the mathematical procedure and the resulting conclusion. They should, however, be able to do exercises such as those on page 130 by rule even if their understanding is incomplete.

Expanded Numerals, p. 130

This is an extension of the pattern introduced in *Book I*, page 26.

A Power of a Power, p. 131

This section is important in that it develops the meaning of a power of a power which provides the basis for the definition of multiplication of integers. A rule is developed for each of the following situations:

1. Page 131-132 with exponents of positive integers or zero.
2. Page 133-135 with one exponent negative.
3. Page 136-137 when both exponents are negative.

These rules are not too difficult for the average student but many will need some assistance. The exercise on page 137 makes use of these rules to simplify expressions with exponents.

Multiplication of Integers, p. 137

Multiplication of integers is not the concept involved in the preceding section. To find powers of powers the additive inverse was used. Now the text uses the ideas developed in powers of powers to define multiplication of integers. These are presented as definitions on page 138 and hence no proof is involved. From this a simply stated rule is given for powers of powers which includes, as special cases, all of the rules mentioned under 1, 2 and 3 above.

Multiplying and Dividing by Powers of 10, p. 140

Pages 140 to 144 develop the short cut methods of multiplying and dividing by powers of ten which are basic to the following sections on scientific notation, metric system of measures and approximation. Lost time and confusion in the exploratory exercise might be avoided by treating the material as teaching lessons with simplified presentation. The section on changing metric units is difficult and might be omitted.

Scientific Notation, p. 143

Approximating with scientific notation is straightforward and practical. However, the exercise on page 157 may prove difficult to some students.

CHAPTER 4: **THE SYSTEM OF RATIONAL NUMBERS** 3 - 4 WEEKS

Optional Content

p. 171	Section 4.2, Let's Explore	p. 172	Section 4.3, Let's Explore
pp. 175-176	Section 4.4, Let's Explore	p. 189	Section 4.13, Let's Explore

The three number systems studied so far have been incomplete in that closure has not been evident under the operations of subtraction and division. The deficiencies of each system, (the system of whole numbers, the number system of arithmetic, and the system of integers) should be explained and related.

In the system of whole numbers, closure in the operations of subtraction and division is not possible since the only numbers having an additive inverse and multiplicative inverse are zero and one respectively.

i.e., $5 - 7 = ?$ and $4 \div 5 = ?$

The number system of arithmetic is not closed under subtraction because all numbers except zero have multiplicative inverses. Closure is exhibited in the operation of division.

In the system of integers, all numbers have additive inverses, hence the set is closed under the operation of subtraction. The set of integers is not closed under the operation of division because only 1 and $^{-}1$ have multiplicative inverses.

The idea of a number system might be reviewed here. Throughout this chapter, the extension of the other number systems into that of the rational numbers should be stressed.

The Set of Rational Numbers, p. 164

The definition of an arithmetic number should be reviewed at the start of the lesson (i.e., an arithmetic number is any number $\frac{a}{b}$ in which "a" can be replaced by any whole number and "b" by any whole number other than zero).

Order and the Number Line, p. 164

After the students have completed Section 4.1, Let's Explore, it might be well to point out that many names might be given to a rational number and written under the points associated with the rational number, i.e. $\frac{^{-}1}{2}$, $\frac{^{-}2}{8}$, $\frac{^{-}14}{56}$, $^{-}.25$.

Addition and Subtraction, p. 167

The integers are in the set of rational numbers and the definition of addition already made for the integers will have to be followed when defining addition for the rationals. The topic provides for pupil participation in the making of definitions. Encouragement of student initiative in these matters is time well spent.

Subtraction, p. 169

As with addition, examples of subtraction in the set of integers and the definition of subtraction involving the addition of an additive inverse could first be obtained from the students. For consistency, the definition of subtraction in the set of rationals must parallel that made for the set of integers.

Multiplication, p. 170

With average and with lower achieving classes, the first approach to the topic mentioned in the teacher's advisory notes on page 170 is recommended. Section 4.4, Let's Explore, p. 175 is an optional exercise but a useful one for reminding students of the definition and properties of an operation. The exercise is needed to establish that, with the existing definition, the properties of multiplication are inherent.

Multiplicative Inverses and Division, p. 177

Students may need to be reminded about the meaning of multiplicative inverse and the definition of division. The second sentence at the top of page 179: "To divide a rational number by another, we multiply by the reciprocal of the second.", is a definition which is needed for the student to solve the questions in Section 4.7, Let's Explore, p. 179.

Exponents and Square Roots, p. 185

Following Section 4.11, Let's Explore, the difference between $^{-}a^{b}$ and $(^{-}a)^{b}$ should be pointed out using examples. Note also on p. 187 the important agreement that $\sqrt{9}$ means 3 and not $^{-}3$, $^{-}9$ means the additive inverse of 9.

The Distributive Property, p. 188

All students should try Exercise 1 in 4.13, Let's Explore. Questions 2 through 7, 4.13, Let's Explore, are optional. The entire set of exercises which follows the display fact should be tried by each student.

More About Fractional Symbols and Division, p. 191

Students should discover that a fractional symbol with one additive inverse sign names the same number no matter where the sign is on the symbol. Before doing the exercise on p. 195, drill the class using examples and discuss the renaming of symbols having an odd or even number of signs.

Betweenness and the Number Line, p. 198

This concept is an important one and a full understanding is required for the topic of graphs which follows. Teachers might well read over pp. 258-259 in *Book I* where the topic was first introduced. The approach is intuitive and no attempt is made to define betweenness.

Symbols for Sets of Rational Numbers, p. 199

Since there are an infinite number of rational numbers between $^-3$ and $\frac{1}{2}$ (the order is dense), there is a need for a standard way of naming all such numbers. The vertical bar means "such that" and the braces indicate that a set is being referred to.

Graphs on a Plane, pp. 200-205

Let the students discover for themselves that the points often form a straight line (See teacher's advisory notes, *Teacher's Edition,* p. 203) or what appears to be a straight line. Avoid using a straight edge on the blackboard with these points and avoid giving advice to students on how to identify sentences whose graphs will be straight lines.

The one example that the students experience in Section 4.19, Let's Explore may not be enough, particularly for average students. Discuss the results with the students as it is done on p. 202, emphasizing that a line separates a plane into 3 subsets: the line itself and two half-planes. Sentences that express equalities have graphs that appear to be straight lines. Sentences that express inequalities have graphs that appear to be half-planes. Graphs of sentences that express equalities appear to be the boundary of the half-plane that is the graph of the corresponding inequality. Two or more points should be used to decide which half-plane is required.

CHAPTER 5: **GEOMETRY: AREAS, REGULAR POLYGONS AND CIRCLE GRAPHS**

3 - 4 WEEKS

The distinction is made between the name of a geometric figure and the name of its measure. A geometric figure is a set of points; its measure is a number. A line segment is a set of points while its measure, a number, is referred to as its length. A region, the interior of a simple closed curve, is a set of points but its measure is called its area.

Segments, p. 217

It is suggested that the rate method be used in solving the problem in the Exercises,number 1. Refer to the *Interim Curriculum Guide, Grade VII Mathematics,* Chapter 8 for details.

Sweeps, p. 219

This method of deriving formulas is simple and easy, and is employed throughout the chapter. It is advisable, however, to show other methods and students should not be given the impression that the "sweep" method is the only way of obtaining the formula.

Trapezoids, p. 221

When the parallelogram formula, p. 227, has been derived, the alternative method of deriving the trapezoid formula as mentioned in the teacher's advisory notes, *Teacher's Edition,* p. 222, should be shown.

Triangles, p. 222

The formula can also be derived by considering the triangles as a special kind of a trapezoid, one of whose parallel sides has the length zero. See also teacher's advisory notes, *Teacher's Edition,* p. 224 for a third method.

Parallelograms, p. 227

Two familiar alternative derivations might be presented, one considering the parallelogram if cut along a diagonal as 2 triangles and further, by dissecting it and repositioning the pieces to form a rectangle. Another method is to treat the parallelograms as a special kind of a trapezoid whose parallel sides are equal in length.

Circles, p. 229

The formula can also be derived by imagining the circle cut along radii into thin congruent slivers which approximate triangles. By meshing half of the slivers with the other half, a figure approximating a rectangle is formed.

The Pythagorean Property of Right Triangles, p. 232

Slower students will require time and much help to complete and understand Section 5.10, Let's Explore. A discussion and review of the material will be necessary to complete the exercise.

Circle Graphs, pp. 240-244

Following Section 5.15, Let's Explore, most students will require more examples developed step by step as in the exploratory exercise. The calculations themselves will present some difficulty to slower students.

CHAPTER 6: **MORE ABOUT INTEGERS AND RATIONAL NUMBERS**

3 - 4 WEEKS

Optional Content

p. 256 Exercises, number 3

Topics related to the rational numbers introduced in Chapter 4 are used in Chapter 6. A review of the extension of the rational numbers to the integers might be done at this time to ensure that the operations and their properties are remembered. The definition of equivalent phrases made in Chapter 1 must be modified now that fractional phrases are to be considered.

In Chapter 4 the rational number system was taught as a mathematical system without the use of physical applications. In Chapter 6 lever problems are posed to show the need for the rational numbers in applications of mathematics to science and engineering.

Applying the System of Rational Numbers, p. 251

The law of the lever is a difficult topic particularly if it is presented without laboratory demonstrations. Several examples need to be taken in addition to those discussed in the texts and three or four periods may be required. The teacher might present each problem orally in order to assist the student to gain a better understanding of the conditions involved in the problem. Some students find it difficult to understand problems unless an oral presentation is made by the teacher.

Factoring Integers, p. 256

The differences between factoring natural numbers and factoring integers is described in four ways. In the Exercises, p. 258, extensive well-developed proofs likely will not be obtained from average students but a class discussion would be helpful and instructive.

Number Phrases for Rational Numbers, p. 261
Phrase Sentences, p. 262
Proving Phrases Equivalent, p. 262

Any replacement for the variable in a fractional phrase which makes the denominator zero is called a non-sensible replacement. This point makes it necessary to modify the earlier definition of equivalent number phrases on p. 38.

The term equation in this textbook refers to any sentence which has the symbol "=" for its verb. The sentence may or may not be true. An equation implies that there are two ways of expressing a number. A phrase sentence says that although the two expressions will name the same number for all sensible replacements, the phrases themselves may differ.

Proving phrases equivalent, p. 264 and Section 6.4, Let's Explore will need careful discussion and average students will require much help.

Another Important Property, p. 275

Specific examples and Venn diagrams might be used to explain the use of the words "if and only if" and why this means that the conditional sentence and its converse are both true.

Long Division with Rational Numbers, p. 278

Attention is called to the section on long division in *Exploring Modern Mathematics, Book I,* Chapter 6, pages 269-275. The change to the algorithm used by many adults can be made later when the basic idea of the process has been understood. Students who have used the *Seeing Through Arithmetic* program in the elementary grades may write the quotients at the side of the work.

CHAPTER 7: **SOLVING EQUATIONS** 4 - 5 WEEKS

The general principles for solving equations are developed in this chapter both by an exploratory method of discovery and by a formal method of presentation.

The content on pages 285 through 294 includes the topics listed below.

1. Terms: open sentence (true or false), solution set, solution.
   Students who have had a *Seeing Through Mathematics* program in Grade VII will be acquainted with these terms.
2. Solving equations by trial and guess.
   a) Equations like $x + a = b$
      $x - a = b$ (p. 287)
   b) Equations like $a \cdot x = b$ (p. 290)
   c) Equations like $a \cdot x + b \cdot x = c$ (p. 292)

The students discover the equation-solving principles through the Let's Explore exercises. These exercises should be done mentally with pupils explaining their methods orally. Do only sufficient questions to establish the rule. Exercises in which the two sides of the equation are reversed should also be given.

General Principles for Solving Equations, p. 295-309

1. The Addition Principle, p. 295 $a = b \rightarrow a + c = b + c$
   Teachers are advised to read the comments on pp. 297-300 in the *Teacher's Edition* carefully, and to note:
   a) The difference between principle and property, p. 298.
   b) The use of the additive inverse. Students should practice writing out *ALL* the steps in the process at first.
   c) Checking the "answer" is essential. The solution makes the statemnt (equation) true; but we do not know this is so until we have replaced the variable in the equation with the solution, i.e., until we have shown that the solution satisfies the original equation.
   d) The format used for checking (the vertical line) should emphasize that in checking we work down, not across.

2. Multiplicative Principle, p. 299 $a = b \rightarrow a \cdot c = b \cdot c$
   a) Note that checking here is much more important, since if $c = 0$, $a \cdot c = b \cdot c$ is true even though $a = b$ is false.
   b) Review reciprocals and the multiplication of a rational number by its reciprocal to get the multiplicative identity, 1. Pupils should write out all the steps until they can do this readily. This is important in *Book* 3, in the proofs. The better students may gradually shorten the steps.

c) Note equivalent phrases, p. 303 $^{-}1 \cdot x \equiv {}^{-}x$

$^{-}x \cdot y \equiv x \cdot {}^{-}y$

$-(a \cdot b) \equiv {}^{-}a \cdot b$

Select examples carefully and illustrate the use of these equivalent phrases.

3. Using the Principles Together, p. 305
   a) Pupils should experiment to see which principle (addition or multiplication) they should use first, for frequently the solution is easier for one order than for the other.
   b) Checking is still important.
   c) Teachers should check pupils' work carefully. Slower students need to write out all the steps.
   d) On page 309 note particularly number 29 (the solution is the empty set) and number 30, which is an identity.

4. The Principle of Zero Products, p. 309
   a) Review the property, Chapter 6, pp. 275-277.
   b) Note the factorization on the left-hand side of the equation and the zero on the right-hand side Number 12, page 311, is intended as a challenge for the brighter student.

5. The Review Exercises, pp. 313-315

   Exercise 1 should be left to the end. The other exercises should be done from time to time; they provide a review of concepts studied in previous chapters.

CHAPTER 8: **POLYNOMIALS** 4 - 5 WEEKS

The polynomials are presented as a new mathematical system in which the elements are symbols and the operations are addition and multiplication. A definition of polynomial is developed and the properties of the operations are studied.

The skills for the elementary manipulation of polynomials (addition, subtraction, multiplication, factoring) are developed and practiced. The following outline will point out the significant steps in the content development.

A. Polynomials (in one variable), pp. 318-325

1. "Combining" Similar Terms, p. 319
   The replacement of subtraction signs by addition signs leads to the use of the plus sign between terms and allows "combining" similar terms.
   Use Review Exercise 2, p. 362.

2. Ascending or Descending Order, p. 322
   Note the standard way of writing expressions.

3. Definition of a Polynomial, p. 323
   Use examples to clarify this for the pupils.
   Note: "coefficient" and "degree" of a term.

4. Terms, p. 324
   Monomial, binomial, trinomial.
   Use Review Exercises, 1, 3-6, p. 362

B. Addition of Polynomials, pp. 325-330

1. Addition should be related to "combining" similar terms. The definition need not be stressed. Use practical examples, including those with minus signs.

2. The Properties of Addition for Polynomials, p. 328

   a) This is a good place to review the properties of a mathematical system that apply thus far. These properties are:

      i. closure under addition
      ii. commutative property
      iii. associative property
      iv. distributive property
      v. the additive identity, 0
      vi. the additive inverse.

      Students require practice exercises using the additive inverse since this leads to the operation of subtraction of polynomials.

C. Subtraction of Polynomials, pp. 331-335

1. Briefly review the definition of subtraction in terms of addition of the additive inverse.
2. Have the pupils write out all the steps at first but encourage them to find their own shortcuts. Avoid the old rule "change the sign and add".
3. Adding and Subtracting in Columns: Make sure the proper signs are placed between the terms.

D. Multiplication of Polynomials, pp. 336-342

1. Multiplying two monomials.
   a) Review the properties of exponents. Use some of the exercises from the previous chapter, e.g., p. 314, numbers 8 and 9.
   b) Note carefully the use of parentheses by the pupils.
2. General Definition of Multiplication of Polynomials, p. 339
   The distributive law is used to obtain the definition. This may be done with examples.
3. Properties of Multiplication for Polynomials, p. 341

   Use the Exercises for drill or omit the Exercises if desired. Oral review of the properties should prove valuable.

E. Special Products of Polynomials, pp. 342-348

1. Products of Two Binomials, p. 342
2. "Sum and Difference" multiplied, p. 345
3. Squaring the Binomial, p. 347

Pupils should learn the patterns involved. Use the frames as an aid in memorization. Do as much oral work as possible with the pupils multiplying mentally. Use Review Exercise, p. 343, number 11, as further practice.

F. Factoring Polynomials, pp. 349-361

1. Common Factors, p. 350
2. Factoring Trinomials, p. 352

   If the patterns for multiplication have been learned thoroughly, it is easy to factor certain trinomials. Look for the familiar patterns. The use of tables, (see p. 354, *Teacher's Edition*), aids the slower students in recognizing the pattern, but drill at the chalkboard is very necessary.
3. Differences of Two Squares, p. 357
   Students should memorize the squares of the natural numbers from 1 through 20.
4. Solving Equations by Factoring, p. 360

   Review the principle of zero products and then give extra exercises for drill from previous sections. Review Exercise 13, p. 347, is useful here.

SEEING THROUGH MATHEMATICS, BOOK 2, SPECIAL EDITION

A. General Considerations

1. Teaching Guide

Teaching Guide, Seeing Through Mathematics, Book 2, Special Edition is available as a special publication from W. J. Gage Limited, 1500 Birchmount Road, Scarborough 4, Ontario (list price $1.75). No teacher should attempt to teach the course without having the *Teaching Guide* available for constant reference during day-to-day lesson preparation. For each lesson the overall purpose is outlined, background concepts are presented and teaching suggestions are developed. The introductory remarks presented on pages 6 to 40 provide background information so pertinent that the teacher would be wise to examine them fully before starting to teach the course. "Lesson Notes", pages 46 to 240, give detailed suggestions for each lesson. For these reasons it is essential that the Grade VIII teacher have available, for reference purposes, the text *Seeing Through Mathematics Book 2, Special Edition* and the corresponding *Teaching Guide.*

2. Semi-Programmed Nature

A *Response Verification Book* is available from the publishers to accompany *Seeing Through Mathematics, Book 2, Special Edition.* Expected replies are given to all the questions asked in the lesson development, and to the exercises which occur at the end of each lesson. These answers are accompanied by explanatory comment where necessary. Each teacher should have a copy of the *Response Verification Book* available for use in the preparation of daily lessons. For many lessons it is possible for the teachers to handle the developmental questions and portions of the "On your own" exercises through an oral approach. Such a technique generally permits more rapid coverage of lessons and increases pupil interest, but requires that the teacher be prepared to deal effectively with pupil responses on the spot. The *Response Verification Book* helps the teacher anticipate pupil answers, both correct and erroneous.

Some schools may desire to make copies of the *Response Verification Book* available for student use. Some lessons might be handled by abler students without much teacher help with the *Response Verification Book* used to check answers. Pupils who have been absent, or pupils who have had difficulty with a particular lesson, will find this book most helpful.

3. Minimum Program and Schedule of Units

The authorized program for Grade VIII consists of seven units containing 87 lessons. No two lessons require equal teaching time, but a coverage rate approximating two lessons each three days must be achieved to permit time for extra drill, review, and testing. To give some guidance to teachers handling the course for the first time, the following suggested timetable was prepared by teachers who have taught experimental classes in Grade VIII.

Teachers who are having difficulty maintaining this schedule should note the content outlined below for possible omission. Because the program develops concepts sequentially from lesson to lesson, it is inadvisable for the teacher to attempt any other modifications in either content or order until she has become familiar with the total program. To avoid imposing a hardship on students in subsequent years, teachers must arrange their coverage so that no lessons other than those suggested below are omitted.

Suggested Time Schedule

Unit	Page No.	No. of Lessons	Possible No. of Lesson Deletions	Net No. of Lessons in Minimum Program	Time in Weeks Minimum-Maximum
8	5	11	0	11	4 - 5
9	63	17	3	14	4 - 6
10	148	23	2	21	10 - 13
11	253	11	1	10	3 - 5
12	303	8	3	5	3 - 5
13	347	10	5	5	3 - 5
14	397	7	5	2	1 - 1
				68	28 - 40

Content that may be Omitted by Classes or Individual Students of Moderate Ability

The authorized minimum program for Grade VIII consist of the seven units included in the text with the following material omitted. Where pupils are capable of handling more than this minimum program it is simply necessary to reduce the amount of material omitted. Throughout the book the exercises entitled "Special Challenges" should be used only for those students or classes well advanced in relation to the time schedule suggested above. Note that Unit 13 is optional for an interim period.

Unit 8 - Geometry and Measurement

Lesson 101 — Use of the protractor: displays 7, 8, 9 and 10 and Exercise A through K, pages 22-23.

Lesson 105 — Proof of the property: For any triangle XYZ, if $\overline{XY} \cong \overline{ZY}$, then $\angle X \cong \angle Z$: Displays 8 and 9 and Exercise A through H, pages 40-41.

Unit 9 - Sets of Numbers and Sets of Points

Lesson 115 — Conditions in three variables, complete chapter, pages 92 through 96.

Lesson 118 — Problems involving conditions in three variables, complete chapter, pages 108 through 112.

Lesson 120 — Central angles and arcs, complete chapter, pages 116 through 121.

Unit 10 - The Rational Number System

Lesson 133 — Use of definition of the product of two rational numbers to decide whether the product of two rational numbers is positive, negative or zero: Display 9 and Exercise A through I, page 185, Display 10 and Exercise A through 0, pages 185-186.

Lesson 135 — A proof of the commutative property of addition in the system of rational numbers. Display 2, paragraph following Exercise E, and Exercises F through J, pages 190-1. A proof of the commutative property of multiplication in the system of rational numbers: Display 4, paragraph following Exercise D and Exercise E through G, pages 191-2.

Lesson 136 — A proof of the associative property of addition in the system of rational numbers: Display 3, the paragraph following Exercise K and A through I, pages 193-4.

Lesson 138 — A proof of the identity-element property of addition of rational numbers: the last sentence of the paragraph following Exercise B on page 201. Displays 2 and 5 and Exercise C through F, page 202. A proof of the identity-element property of multiplication of rational numbers: the paragraph following Exercise B and Exercise C, D and E, page 202. A proof of the zero property of multiplication of rational numbers: Display 8, paragraph before Exercise 0 and Exercise 0, P and Q, page 203.

Lesson 145 — Integral powers and scientific notation. Complete chapter pages 233 through 237.

Unit 11 - The Real Numbers

Lesson 157 — Solution sets of conditions in three variables: Display 7 and Exercise A through I, page 290.

Lesson 159 — Problems involving conditions in three variables: Display 3 and Exercise A through K, page 297, Display 3 and 4 and Exercise I through R, page 298.

Unit 12 - Measurement

Lesson 162 — Scientific notation as used to indicate the number of significant digits in a numeral: The paragraph preceding Exercise A, page 317, and Exercise A through G and Display 4, page 317.

Lesson 165 — Trigonometric rate pairs. Complete chapter, pages 330-3.

Lesson 166 — Using a table of tangents, sines and cosines. Complete chapter, pages 334-7.

Lesson 167 — Solving problems involving measure by using trigonometric rate pairs: Display 3 and 4 and Exercise A, B and C, page 339; pages 340 and 341, Exercise X, Y and Z, page 342.

Unit 13 - Formulas (This whole unit is optional for an interim period.)

Lesson 169 — Relations determined by $y = kx^2$. Complete chapter, pages 350-3.

Lesson 170 — Relations determined by $y = \frac{k}{x}$: Exercises H through K, page 354; "On Your Own" Exercise 3, and last two parts of Exercise 4, page 357.

Lesson 171 — Relations determined by $y = \frac{k}{x^2}$. Complete chapter, pages 358-9.

Lesson 172 — Relations determined by conditions for variation: Complete chapter, pages 360-6.

Lesson 173 — Problems involving conditions for variation of the types $y = kx^2$ and $y = \frac{k}{x^2}$ and the part of the lesson that deals with properties of squares and square roots: Display 3 and last three paragraphs, page 367; page 368; Exercise D through H, page 369; Display 8 and last two paragraphs, page 369; Exercise 0 through W, page 370.

Lesson 174 — Trigonometric formulas: the last paragraph, page 374; Display 5 and 6 and Exercise A through L, page 375.

Lesson 175 — Using formulas involving trigonometric rate pairs to solve problems; Exercise A, through 378 and the paragraph preceding this Exercise; Display 6 and Exercise B through K on page 379.

Unit 14 - Probability and Statistics

Lesson 178 — Events and their probabilities. Complete chapter, pages 397 through 401.
Lesson 179 — Probabilities in a sample space Complete chapter, pages 401 through 406.
Lesson 180 — Mutually exclusive events: Complete chapter, pages 406 through 411.
Lesson 181 — Independent events. Complete chapter, pages 411 through 415.
Lesson 182 — Empirical probability. Complete chapter, pages 415 through 420.

B. Lesson Comments

Unit 8 - Geometry and Measurement

The Grade VIII S.T.M. course begins with a study of geometry. The geometry presented in Grade VII was nonmetric; that is, it involved no numbers. In this first unit of Grade VIII the theme is measure. All eleven lessons deal with metric geometry.

Lessons 98-100 — Measure of a Segment, Practical Measurement, Problems Involving Standard Units of Length

These three lessons form a unit, and should be studied by the teacher in a block before teaching is begun. Some teachers present lessons 98 and 99 together. It is wise to begin this section with a review of ideas already developed concerning segments. Review the correspondence between the rational numbers of arithmetic and the points in a number line, the coordinates of a point, and the meaning of congruence, less than, and greater than for segments. Be sure that the meaning of the measure of a segment as presented in Lesson 98 is made quite clear. This understanding is of fundamental importance to subsequent lessons. Introduce the idea of measuring a segment by comparing it with a unit segment, through the use of classroom objects. For example, different measures of one edge of the teacher's desk will be found if pieces of chalk of different lengths are used to represent unit segments.

Once students understand that different unit segments produce different measures of the same segment, the need for standard units of measure becomes obvious. Note that although it is impossible to measure a physical object without error, it is assumed throughout Lesson 99 that it is possible to find the exact measure of an object by comparing the length of the object with a standard unit. The section on finding the sums and differences of measures on pages 10 and 11 provides an excellent opportunity to review the addition and subtraction of the rational numbers of arithmetic.

The problems in Lesson 100 involving standard units of length are solved using the rate pair approach. Use this approach consistently, even though many students can find the solution more directly, since the approach provides a firm foundation from which to attack more difficult problems. Students can be expected to memorize the tables on pages 14 and 16, since many of the comparisons between units are already familiar to them. The teacher should undertake to make the standard units under discussion real to the students. Standard metric units are usually available from science supplies, for example. The "Applying Mathematics" section on pages 16 and 17 should be completed carefully.

Lesson 101 — Measure of an Angle

The approach to the measure of an angle is quite different from that usually found in traditional texts. The measure of an angle, like the measure of a segment, can be found using any convenient unit measure. In finding the measure of an angle, the unit angle most commonly used is the one that makes 90 the measure of a right angle. Unit angles other than the degree are sometimes used. See page 51 of the *Teaching Guide*.

Students should be given a good deal of practice in the use of the protractor to find an approximation to the degree measure of an angle. Refer to the comments in the *Teaching Guide*, page 51. Notice that by definition, an angle cannot have a measure of zero degrees or one hundred eighty degrees. Students should memorize the definition on page 21.

Lesson 102 — Angles Formed by Two Coplanar Lines

Lesson 102 makes use of a number of the ideas presented in Lessons 26 through 29 of the Grade VII course. It would be wise to begin with a review of the major ideas and definitions presented in these lessons. The content overview on pages 53 and 54 of the *Teaching Guide* provides valuable background for this lesson.

Lesson 103 — Angles Formed by Three Coplanar Lines

Students develop a better understanding of coplanar lines when they are first demonstrated in space, using rulers or pointers to represent the lines. Through this type of demonstration the pupil can see that any two intersecting lines are members of one and only one plane, and hence are coplanar. Develop an intuitive understanding of "adjacent angles" and "corresponding angles" through the use of a number of examples before attempting the more formal definitions. Coloured chalk, construction paper, drinking straws, peg-boards and golf tees, overhead projector transparencies and many other visual and manipulative materials can be used very effectively in the development of lessons of this type. Students should develop a clear understanding of the meaning of "converse" as it is presented on page 31.

Lesson 104 — Measures Associated with Triangles

Review with the students the definitions of scalene, isosceles, and equilateral triangles given on page 101 in S.T.M.I. The section on the classification of triangles by angles is very similar to that found in traditional texts. Good suggestions for teaching that the sum of the measures of the angles of a triangle in degrees is 180 are given on pages 57 and 58 of the *Teaching Guide*. Do not be too concerned if slower students do not follow the proof given on page 34.

Lesson 105 — Congruent Triangles

It is advisable to begin this lesson with a review of one-to-one correspondence and the definition of congruent triangles given on page 111, S.T.M.I. Students should be able to demonstrate the six different ways that the vertices and hence the sides and angles of two triangles can be placed in one-to-one correspondence. One way to demonstrate these correspondences is to prepare large representations of triangles from cardboard or construction paper. It is suggested that the following pairs be produced: 2 non-congruent scalene triangles, 2 congruent scalene triangles, 2 congruent isosceles triangles, 2 congruent equilateral triangles. Three of the correspondences can be demonstrated by holding one of the triangles steady while the other is rotated. The other three are obtained by flipping one of the triangles before rotating. Large letters should be printed on each side of the triangle representations. Masking tape can be used to join the vertices after each rotation It helps if each correspondence is listed on the blackboard as it is discovered. Students usually grasp the ideas of congruence correspondence and corresponding sides and angles when demonstrations such as these are used. The num-

ber of congruence correspondences between congruent scalene, congruent isosceles, and congruent equilateral triangles is easily demonstrated.

Lesson 106 — The Quadrilateral

This lesson introduces students in a more formal way to some of the characteristics of quadrilaterals. Since all geometric figures are sets of points, the language of sets is used to describe these characteristics. Peg-board, golf tees and brightly coloured strings are useful visual aids. Figures cut from construction paper have the disadvantage that they often represent the interior of the figure as well as the figure itself, and therefore leave a false impression with the pupil.

The *Teaching Guide* offers suggestions for showing the important relationships among the set of all simple polygons, the set of all quadrilaterals, the set of all parallelograms, the set of all rectangles, and the set of all squares. See page 61 for a Venn diagram representing these relationships.

Lessons 107-108 — Area of a Parallelogram, Area of a Triangle

These two lessons will probably require at least three class periods to complete. The teacher should be completely familiar with the equivalent rate pair approach to finding area before starting to teach the lesson. It is wise to stay with this approach throughout. Students who studied S.T.A. in Grade VI will be familiar with it. Formulas are developed later.

Note that, strictly speaking, a quadrilateral does not have an area. One finds the area of a "rectangular region", for example, not the area of a rectangle. Students should develop definitions for such words as "altitude" and "base" only after the ideas have been grasped intuitively from a number of examples.

The students can be expected to memorize the table of area measures on page 57 of the text. The "Applying Mathematics" problems on pages 57 and 58 should be studied in detail. This study may require at least two additional class periods.

Unit 9 - Sets of Numbers and Sets of Points

Unit nine reviews and extends the ideas developed in the previous eight units. The unit can be divided into two parts. Lessons 109 through 118 are concerned mainly with compound conditions and problems involving such conditions. Lessons 119 through 125 are concerned with introducing new geometric figures, with constructing geometric figures and with finding areas of geometric figures.

Lesson 109 — Compound Conditions That Include the Connective "and".

This is a review lesson. Several reminders, which should be reviewed thoroughly with the class before any attempt is made to teach new concepts, make up the bulk of the lesson. The students are introduced to interval notation,

e.g., $\{x \mid \frac{11}{4} < x \wedge x < \frac{17}{4}\}$ becomes $\{x \mid \frac{11}{4} < x < \frac{17}{4}\}$

Lesson 110 — Compound Conditions That Include the Connective "or".

Lesson 110 introduces a new connective, "or", and distinguishes it from the now familiar "and" connective. Be sure students understand that "or" in the mathematical sense means "one or the other or both", and is related to the union of sets. The symbol is the reverse of the "and" symbol. The symbol for "or" is written "$\vee$".

Certain compound conditions involving the connective "or" can be shortened by the use of symbols for "is less than or equal to" ($\leqq$) and "is greater than or equal to" ($\geqq$).

The graph work on page 72 should provide the pupil with a visual representation of the ideas and symbols he has learned earlier in the lesson. The symbols used in the graphs should be reviewed carefully.

Many students will require considerable assistance with the "On Your Own" exercises, page 73.

Lesson 111 — Compound Conditions in Two Variables

The first half of this lesson is a review of the procedures for finding the solution sets of conditions in two variables that include the connective "and", and of the procedures for graphing these solution sets. Refer to the *Teaching Guide,* pages 76-77. This review should be carefully planned and carried out in detail. It will probably require at least one class period.

The second half of the lesson introduces procedures for finding the solution sets of compound conditions in two variables that include the connective "or". Students usually do not have too much difficulty with this section if the review of conditions with the connective "and" has been conducted well.

Lesson 112 — The Complement of a Set

Conditions involving the connective "not" are introduced in this lesson. The symbol is the same as that used for equivalence, " $\sim$ ". In order to find the solution set of such conditions the student must understand the idea of the complement of a set. This idea is introduced first in the lesson. Since some students find the ideas presented in this lesson rather confusing, the lesson should be taught using a large number of examples. Physical examples using sets of objects in the classroom, including the students themselves, have been found helpful in teaching the idea of complement.

Lessons 113-114 — Intervals, Conditions Involving the Idea of "approximately equal to".

Some teachers have found it possible to combine these two lessons into one, especially if the students have learned their previous lessons well. Examples of the notation used for open interval, half-open interval and closed interval follow:

open interval $\{ x \mid 3 < x < 5 \}$
half-open interval $\{ x \mid 3 < x \leqq 5 \}$
closed interval $\{ x \mid 3 \leqq x \leqq 5 \}$

The symbol for "it is not the case that" may also be used. For example,

$\{x \mid \sim(x < 3)\} \quad = \quad \{x \mid x \geqq 3\}$

The idea of tolerance is most important and should be taught thoroughly. It forms the foundation for the lessons in Unit 12 that deal with practical measurement.

Lesson 115 — Conditions in Three Variables

This lesson is to be omitted except for above average classes.

Lessons 116-117 — Problems Involving Conditions in One and Two Variables, Problems Involving Conditions for Equivalence

The problems in these two lessons give the students an opportunity to develop conditions similar to those studied earlier in this unit. Since problems of this type cause considerable difficulty to a number of students the teacher should be prepared to provide more than the usual amount of individual attention. Teachers have found it helpful to provide students with complete model answers to type problems. Students should be expected to define their placeholders and to show each step leading to their solution sets and answers.

Before assigning the "On Your Own" exercises on pages 106-107, the teacher would be well advised to review with the class terms such as discount, commission, per cent of increase and decrease, and regular price.

Lesson 118 — Problems Involving Conditions in Three Variables

This lesson is to be omitted except for above average classes.

This marks the end of the first division of this unit. Testing could be carried out at this point.

Lesson 119 — Intersections of Lines and Circles

Much of this lesson is review. Since some students will have experienced difficulty in Grade VII applying ideas of condition, universe, and solution set to sets of points, review in this area should be carried out carefully. Check the *Teaching Guide,* page 95, for excellent suggestions for providing for individual differences. Students will experience little difficulty grasping the ideas associated with tangents and secants.

Lesson 120 — Central Angles and Arcs

This lesson is not included in the minimum program. However, it is recommended that it be taught if time permits. Students enjoy the lesson, especially if interesting visual aids are employed.

Lesson 121 — Polygons

Lesson 121 can be completed in one class period. Much of the lesson is review. The names and descriptions of four additional polygons are introduced, along with the symbols for "perpendicular to" and "parallel to". Note that the symbol for "parallel to" is slightly different from the one in previous common use.

Lesson 122 — Triangles and Related Segments

The concepts included in this lesson can be taught in one class period. It has been found advantageous, during the review of ways in which triangles can be identified, to have the children draw representations of the different kinds of triangles under discussion. Some teachers incorporate much of the work on construction covered in Lesson 123 into this lesson by having students construct bisectors of angles of a triangle, midpoints of a segment, and medians of triangles.

Lesson 123 — Geometric Constructions

This lesson may require two class periods because most students will be quite unfamiliar with geometric instruments and their use. It may be necessary to begin with a review of the names of the instruments. Students should be made aware of the restrictions placed on the use of the tools and procedures in making constructions. These are listed on pages 102-103 of the *Teaching Guide*. The constructions learned in this lesson are used in subsequent lessons.

Lesson 124 — Congruent Triangles

This lesson may take two class periods to complete. The students should be allowed to construct the triangles themselves. As the *Teaching Guide* suggests, some pupils will grasp the three very important properties as they carry out the constructions. Since pupils have had little experience in the use of geometric instruments they will likely require considerable individual attention.

Lesson 125 — Areas of Geometric Figures

If Lesson 108 has been taught thoroughly, students will have little difficulty with this section. Note that in this lesson conditions for equivalence are still used to find the areas of geometric figures. In Lesson 177 the common formulas are derived. It is not advisable to introduce the formulas in this lesson. The greatest difficulty students have is developing an understanding of why 2x is used in the condition for finding the area of a triangle.

Unit 10 - The Rational Number System

Unit 10 develops the set of rational numbers. Since the approach to what have traditionally been called "signed numbers" may be quite new, the teacher is well advised to become thoroughly acquainted with the total unit before attempting to teach it. The approach used is a good one if the teacher is well prepared. The *Teaching Guide* provides valuable assistance during lesson preparation. Pages 17-23 of the *Guide* present an overview of the total unit.

This unit is the longest in the Grade VIII course. It is suggested that it be broken into sections as follows:

Section 1 — Lessons 126-133
Section 2 — Lessons 134-137
Section 3 — Lessons 138-142
Section 4 — Lessons 143-145
Section 5 — Lessons 146-148

Testing may be carried out at the end of each section.

Lesson 126 — The Natural Numbers and the Rational Numbers of Arithmetic

This lesson reviews some of the properties of N and R_a, which were presented in Units 6 and 7 of the Grade VII program. Teachers not familiar with these properties, especially teachers who have not taught from the Grade VII text of this series, should review these two units. Motivation for the extension of R_a is provided if pupils are reminded that subtraction is not always possible with the rational numbers of arithmetic.

Lesson 127 — The Positive Rational Numbers

The average class will require at least two periods for this lesson. In contrast to many other modern texts, the approach to the positive rationals is through sets of equivalent ordered pairs. Note that the arrowhead used in the symbol for directed segment always points to the right, while in graphs the arrowhead may point in either direction. (See the *Teaching Guide,* page 113.)

The teacher should study carefully the meaning of equivalent directed segments as outlined in the *Teaching Guide,* pages 12-15. Note, for example, that segments with the same measure may or may not be equivalent.

It is suggested that the ideas associated with the definition of positive rational numbers and with the standard name be developed through student participation, rather than through a formal presentation by the teacher.

Suggestions for providing for individual differences are given on page 16 of the *Teaching Guide.* "On Your Own" exercises 1-17 may be answered orally. The teacher may wish to provide extra exercises similar to these. Exercises 18-26 could be assigned as homework.

Lesson 128 — The Negative Rational Numbers; The Set of Rational Numbers

Most average classes will require two class periods for this lesson. Some teachers have found it helpful to employ grouping within the class. Better students could work as a group, freeing the teacher to develop the lesson more slowly with the less able.

After the definition of negative rational numbers has been developed, the students learn that the set of rational numbers (R) consists of the positive rational numbers, the negative rational numbers and zero. Study of the rational number line illustrates these ideas and helps the student to organize the concepts into a meaningful pattern.

Lesson 129 — Ordering of the Rational Numbers

Many students experience difficulty in ordering rational numbers, especially negative rationals. The rational number line and practical examples assist pupils to overcome this difficulty. It is important that pupils be given sufficient practice with the formal procedure for determining whether one number is greater than or less than another. (See displays 1 and 2, page 167 of the text.) The teacher should provide further examples for the less able student. These students should be required to follow the step-by-step procedure outlined in the text. The "Instruction Notes" on pages 121-122 of the *Teaching Guide* give excellent suggestions for the practical application of ordering negative numbers.

Lessons 130-131 — Preparation for Addition, Sum of Two Rational Numbers

These two lessons can be treated as one, but the combined material cannot be completed in one period. The teacher should read the provisions for individual differences suggested on pages 25 and 28 of the *Teaching Guide,* especially as they apply to slower learners. It may be necessary to give the students additional practice in adding with ordered pairs before the rules are developed and applied.

Lessons 132-133 — Preparation for Multiplication, Product of Two Rational Numbers

Abler students should be able to complete these lessons in two class periods. See the *Teaching Guide,* pages 131 and 134, for suggestions for slower students. Note that a section of these lessons is not included in the minimum program.

Lesson 133 is the last lesson in the first section. At the end of this section review sheets could be utilized and the first testing of this unit could be carried out. Some teachers use work sheets on this section to provide constant review while subsequent lessons are being presented.

Lesson 134 — The Elements of R_a Related to Certain Elements of R.

It is wise to begin this lesson with a short review of the complement of a set. (See lesson 112, pages 78-81). Students readily grasp the isomorphism between the elements of Ra and Rn.

Lesson 135 — The Commutative Properties

This lesson is the first of a series designed to establish the properties of addition and multiplication required for a number system. Similar properties were developed to establish the natural numbers and the rational numbers of arithmetic as number systems. Teachers have found it helpful to review these properties before beginning this series of lessons.

Note that the proofs in this lesson are not included in the minimum program. Any teacher choosing to present these proofs will find pages 137 and 138 of the *Teaching Guide* most helpful. Teachers who have taught the proofs report a saving of time later in the unit where students are required to justify their chains of equivalent conditions when finding solution sets of conditions.

Lesson 136 — The Associative Properties

Note that the proof in this lesson is not included in the minimum program. If the proof is not taught, the suggestion for slow learners given on pages 140 and 141 should be followed. Note the comment above concerning the saving of time later when proofs are taught at this stage.

Students tend to guess at the first eight qestions of the "On Your Own" exercise. Force them to justify their answers.

Lesson 137 — The Distributive Property and the System of Rational Numbers

Note that the distributive property of multiplication over addition of rational numbers is accepted without proof. The concept should be developed through the use of a number of examples.

The last portion of this lesson is devoted to the study of the rational numbers as a number system. This part of the lesson provides an excellent opportunity for review. Parroting of properties is of little value. An understanding of the properties should be developed. Utilization of the properties in computational short-cuts assists in this understanding.

It may be necessary to introduce brackets in addition to those with which the pupils are already familiar in order to teach this section effectively. A number of the exercises in the "On Your Own" section may be done orally. Students should justify their answers whether they are given orally or on paper.

The "Special Challenge" on page 199 should be reserved for very able students.

Since this lesson is the last in the second section, review and testing could be carried out at this point. "Applying Mathematics", page 200, should not be neglected. The section serves as an excellent review.

Lesson 138 — The Identity Elements

Note that the portions of this lesson associated with proofs are omitted from the minimum program. If these parts are not taught the properties can be developed by showing that the zero properties and the properties of one developed for R_a have counterparts in the rational number system. Since the proofs are not difficult they can be presented to average and above average pupils without too much difficulty.

Lesson 139 — The Additive Inverse and Subtraction

The ideas presented in this lesson and in Lesson 140 are essential to so much of the subsequent work that the teacher should devote sufficient time to ensure complete mastery.

The distinction among the symbols for a negative rational, an additive inverse, and the operation of subtraction, as reviewed in the *Teaching Guide,* page 146, should be studied carefully by the teacher. These distinctions were often ignored in traditional texts. The correct use of brackets is important to the development of pupil understanding.

Some pupils will have forgotten the meaning of the words "minuend", "subtrahend", and "difference". These meanings should be reviewed with the class.

Lesson 140 — The Multiplicative Inverse and Division

Most classes should devote at least two periods to this lesson. The teacher should ensure that every student has a firm grasp of the meaning of "reciprocal" (multiplicative inverse) before proceeding with the remainder of the lesson. Provide more examples than are presented in the text, especially where negative numbers are concerned. Note that the reciprocal property can be expressed in two ways. The teacher may wish to add exercises to those suggested in the text. Pupils must thoroughly grasp the ideas presented in this lesson.

Lesson 141 — Using Mixed Numerals and Decimals for Rational Numbers

This lesson is basically a summary of computational procedures for mixed numerals and decimal numerals that name rational numbers. All students should demonstrate facility with these computational procedures. If the teacher has been maintaining the necessary skills through the periodic use of review sheets, this lesson can be completed in two or three lessons.

It was suggested on page 4 of the Grade VII *Curriculum Guide* that students be encouraged to use the conventional division algorism. The teacher should check to determine the algorism in which her pupils are most proficient. Short division is not presented in this series of texts. Many teachers have found it advantageous to explain the method during their presentation of this lesson.

Lesson 142 — Repeating Decimals as Names of Rational Numbers

It is advisable to review the ideas associated with complete factorization, prime numbers, and powers of numbers. These topics are included in Unit 6 of the Grade VII text.

Note that every rational number can be expressed as a repeating decimal if terminating decimals are considered to have a repetend of zero.

Take time to ensure that every child knows the decimal mates of the most frequently used common fractions.

Lesson 142 is the last lesson in section three. Testing and review could be carried out at this point.

Lesson 143 — Positive Integral Powers and Scientific Notation

In Unit 5 of the Grade VII text students studied the use of exponential notation to express certain natural numbers. A review of this work should be carried out before the material in Lesson 143 is studied. See "Instruction Notes", page 154 of the *Teaching Guide* for other suggestions for review.

It will probably be necessary to give students additional exercises in the use of the product and quotient properties of positive, integral powers. These important ideas should be developed thoroughly. The work with scientific notation helps to reinforce what the students have learned about these properties. See the *Teaching Guide,* page 154, for specific examples of the use of scientific notation.

Lesson 144 — Negative Integral Powers and the Zero Power

This lesson contains a number of important, sometimes difficult, ideas. An average class should be expected to take two class periods to master the lesson. The teacher will likely find it necessary to provide extra practice sheets. It is essential that students be able to find the sum and difference of positive and negative integers quickly and accurately before this lesson is undertaken.

Lesson 145 — Integral Powers and Scientific Notation

This lesson is not included in the minimum program. Teachers who choose to teach it should refer to the *Teaching Guide,* pages 156-158.

This marks the end of the fourth section. Testing and review are suggested.

Lesson 146 — Equivalent Conditions

Lessons 146-148 have been set aside in a separate section because the material is so important to subsequent work. Now that students have been introduced to the set of rational numbers and the properties of the rational numbers, they are ready to develop a more sophisticated and formal approach for finding solution sets of conditions.

Lesson 146 may require considerable time, especially if students have not been introduced previously to the idea of justifying each step in a solution. (The proofs omitted from the minimum program earlier require similar patterns of thought.) Read the study guide carefully before attempting to teach this lesson. The suggestion that the teacher might prepare and display a list of definitions and properties that are particularly relevant to this lesson is an excellent one. Displays of chains of conditions developed through the utilization of these definitions and properties might also be prepared.

Students will require more practice than the text provides. Teachers have found it advisable to prepare extra practice sheets for each new type of condition introduced. Pupils should be required to justify each step in their solutions.

Lesson 147 — Inequalities and Compound Conditions

This lesson, like Lesson 146, may require considerable time to complete, because it includes difficult and important ideas. Notice that the solution sets for conditions for inequality are found by relating these conditions to conditions for equality. This procedure is dropped later as other properties are introduced.

Students will need considerable practice in finding solutions of compound conditions and in graphing solution sets of conditions for inequality. The teacher will probably find it necessary to prepare exercises in addition to those given in the text.

Lesson 148 — Conditions for Problems Involving Rational Numbers

The problems in this lesson provide an opportunity for the student to employ his recently learned procedures to find answers to practical and abstract numerical problems. Many students will require assistance in developing conditions from the problems. As in the previous lesson, the pupils can be expected to justify each condition in the chain leading to the solution set. The *Response Verification Book* can be used as a guide for selecting problems for the class, especially if Lesson 145 has been omitted.

This is the end of Unit 10. Review and a unit test could be used here.

Unit 11 - The Real Numbers

Unit Eleven which introduces the last of the number systems studied in Grade VIII, contains eleven lessons. Most of the ideas presented will be familiar to the teacher, but the approaches used to develop these ideas are quite different from those used in traditional courses. Teachers of experimental classes report that attempts to use traditional approaches usually lead to poor understanding and later difficulties. The teacher should be thoroughly familiar with the type of development used in the text before beginning to teach the unit.

Lessons 149-150 — Squares and Square Roots, Determining Square Roots

The ideas of the square of a number and the square root of a number developed in these two lessons are important because they are useful in problem solving, and because they indicate the need to develop a number system that is an extension of the rational number system.

The three important properties of squares developed in Lesson 149 are easily learned. The properties should not be expressed in formal mathematical language until after the concepts are clearly developed through the use of a number of examples. Some students will have difficulty distinguishing between the square and the square root of a number.

Note that the common symbol for square root indicates only the principal square root. Squares of fractional and negative numbers and square roots of fractional numbers will require special attention. Some teachers require their students to develop and memorize the squares of numbers from one to twenty.

The methods used in Lesson 150 to determine square roots are designed to develop an understanding of the process. Teachers unfamiliar with these procedures will need considerable practice in them before attempting to teach the lesson. Do not ask students to find the square root of large numbers through the use of these procedures. The important thing is to develop understanding. The traditional method of finding square roots outlined in the *Teaching Guide* is more efficient with large numbers, but is difficult to teach with understanding. This method can be taught after students have a thorough grasp of the method outlined in the text. A table of squares and square roots is found on page 461 of the text.

Lessons 151-153 — The Real Numbers, Algebraic Properties of the Real Numbers, Order and Completeness Properties

In Unit 10, students studied numbers that could be expressed as repeating infinite decimals, the rationals. The work with square roots in this unit indicates that there is also a set of numbers that can be expressed as non-repeating infinite decimals, the irrationals. The real numbers are introduced by considering the union of these two disjoint sets. It is important that the relationships among the rationals, irrationals and real numbers be taught thoroughly. Work with the real number line helps to clarify these relationships.

The teacher would do well to study and prepare Lessons 152-155 as a group. The algebraic properties discussed in Lesson 152 are essential to the development of Lesson 154, and the three properties of "less than" developed in Lesson 153 are essential to the development of Lesson 155. It is important, therefore, that students be led to a thorough understanding of all these properties in order that they can identify and use them in specific situations. The teacher will find it helpful to commit the properties and their names to memory before attempting to teach these lessons.

Lessons 154-155 — Solution Sets of Conditions for Equality, Solution Sets of Conditions for Inequality

Experience has shown that these two lessons require more time than is suggested in the *Teaching Guide*. Their combined content is almost equivalent to all the work on equations presented in the former Grade IX text. Lesson 154 reviews and extends the work covered in Lesson 146. It is advisable to review Lessons 139, 140 and 146 before beginning to teach Lessons 154 and 155.

It is a mistake to fall back on the more familiar approaches used in traditional courses. Students seem to have little difficulty following the meaningful procedures suggested in this text, if the teacher is well prepared to teach the lesson. Students may require more practice than is provided by the exercise on page 280. Students' work should be examined carefully to determine whether each link in the chain of conditions has been justified. Short-cuts should not be allowed at first.

In Lesson 15 students use a new approach to finding solutions sets of conditions for inequality. Previously, students solved a condition for inequality by first finding the solution set of a related condition for equality. Now, since they are equipped with the order properties introduced in Lesson 153, students can solve these conditions by developing a chain of equivalent conditions. Students need a good deal of practice in developing chains of equivalent conditions. The teacher should work very carefully through a number of examples. The work to the bottom of the first column on page 282 may take two full class periods, and the material on compound conditions included in the remainder of the lesson may take another two periods. It is advisable to prepare extra practice sheets in advance.

Lesson 156 — Coordinate Axes and the Real Plane

In this lesson students learn to associate points in a plane with ordered pairs of real numbers. The new vocabulary introduced in this lesson should be learned thoroughly. Since students have not had experience with graphing Cartesian sets for some time, it has been found valuable to introduce this lesson with a review of the work on page 75 of the text. Some good suggestions for providing drill in locating points associated with ordered pairs of real numbers are given in the *Teaching Guide*. A large blackboard graph grid for use by the teacher and graph sheets for use by the students are valuable aids to the presentation of this lesson. Teachers have found that an overhead projector is a valuable aid in this graph work. Transparencies showing a graph grid are available from commercial firms or can be prepared.

Lesson 157 — Conditions in Two and Three Variables

This lesson contains a number of important, and sometimes difficult, ideas. More time than that suggested by the guidebook will be needed if students are to gain a thorough grasp of the ideas presented. The work included on pages 287 and 288, supplemented by additional examples, will probably require at least one class period, since it involves preparing charts and graphs for both conditions of equality and conditions for inequality. Exercises A through M on pages 288, 289 and 290 develop the graphical and algebraic methods of finding the solu-

tion sets of compound conditions involving linear conditions for equality. The teacher should expect to spend at least two lessons on this section. Again, extra examples will be necessary to develop the ideas thoroughly. Note that the section concerned with finding the solution set of a compound condition in three variables is optional. This section provides excellent supplementary material for more able students.

Lesson 158 — Problems Involving Conditions in One Variable

The problems in this lesson are little different from others that the pupils have done earlier. However, since the universe is now the set of real numbers, it is often necessary to develop a compound condition for a problem when previously a simple condition would adequately describe the situation. Teachers will find a study of the content overview in the *Teaching Guide* helpful in their preparation for the teaching of this lesson. Some students will need extra help in developing chains of equivalent conditions and in expressing solution sets.

Lesson 159 — Problems Involving Conditions in Two and Three Variables

The problems in this section are considerably more difficult than most problems that students have met previously. It has been found advisable to develop a number of examples with the class before the students are asked to work the problems alone. Students will require extra help in developing chains of equivalent conditions when two variables and compound conditons are involved. Note that exercises A to R on pages 297 and 298 are optional. If the more able students have studied the optional sections of Lesson 157, the work with problems involving compound conditions in three variables provides excellent supplementary material.

Unit 12 - Measurement

This unit consists of eight lessons. Two of these lessons, Lessons 165 and 166, are omitted entirely from the minimum program. Most of Lesson 167 is also omitted. These lessons include work in trigonometry. The lessons required in the minimum program include additional study of standard units, including the metric system, work with greatest possible error, relative error, precision and accuracy, and an introduction to similar triangles and problems involving the use of indirect measurement. All of these areas are important to further studies in mathematics.

This unit treats the topic of of measurement in such a way that the vagueness that surrounded such ideas as computation with "denominate numbers" in the past has been removed by careful attention to ideas of precision and accuracy and their relation to sums and products of measures. (p. 188 *Teaching Guide, S.T.M.2*).

In Lesson 160, measurement is treated as though it were possible to find the exact measure of a physical object. In the succeeding lessons of this unit it is acknowledged that the exact measure of the physical object cannot be found.

Lesson 160 — Standard Units of Length

The first part of this lesson reviews some of the ideas introduced in Lessons 98 and 99. The teacher may wish to make reference to these units. The lesson will probably require two class periods. It has been found helpful to give students a "feeling" for the lengths of the standard units of the metric system by having them measure objects using metric units. They can measure their height in centimetres, for example; or measure the width of a strip of film. (35, 16 and 8 mm. film is easily obtained by students.) Students develop a better understanding of the table in display 5, page 306, if they are asked to determine the first three relationships using measurement rules. They can be expected to memorize this table and the prefixes of the metric system of measure.

Most students will require considerable assistance in setting up the conditions for equivalence necessary to carry out conversions, especially where compound conditions are involved. See the suggested provisions for individual differences, page 190.

Note the references in the *Teaching Guide,* page 190, to "Applying Mathematics".

Lesson 161 — Greatest Possible Error In Measurement

Stress the difference between mathematical and physical measures. Teach carefully the relationship between the greatest possible error and the unit of measure used. (The greatest possible error is one-half of the unit of measure used.)

The teaching of error in measurement involves the ideas of closed interval and tolerance. It is wise to review Lessons 113 and 114 with the class before proceeding too far with this lesson.

Give plenty of oral work to ensure that pupils develop a good understanding of the symbols used to express physical measures. Explain that mathematicians have agreed to express physical measures in such a way that is obvious what unit of measure is being used. In a decimal numeral for a physical measure the digit farthest to the right indicates the unit used to obtain the measure. A physical measure expressed as 210 ft. $\pm$ 5 indicates that ten feet is the unit of measure being used.

Be sure that students grasp the idea that a physical measure determines a closed interval, and that the mathematical measure of the object is a member of that interval.

This lesson will require at least two class periods.

Lesson 162 — Precision and Accuracy

Note that the portion of this lesson dealing with the use of scientific notation is not included in the minimum program (page 317).

The teacher is advised to read Lesson 163 before starting Lesson 162. One teaching period can be saved by integrating Lessons 162 and 163. After the meaning of "precision" has been taught from Lesson 162, the concepts can be applied directly to the addition and subtraction of physical measures, Lesson 163. In the same way "accuracy" can be applied directly to the multiplication and division of physical measures.

Be sure that students understand the distinction between "precision" and "accuracy". Greatest possible error is used to compare the precision of measures. Relative error (the error per one standard unit of measure) is used to compare the accuracy of measures.

Some time should be spent to develop a clear understanding of what is meant by significant digits. The physical measures with the most significant digits are the most accurate. The rule for finding good approximations of the products and quotients of physical measures by rounding to the correct number of significant digits is given in Lesson 163.

The section not included in the minimum program may be taught if the class has studied Lesson 145, Unit 10. The work with scientific notation helps further the understanding of significant figures.

Lesson 163 — Computations Involving Approximations

The rules for computation with numerals that express physical measures should be developed with understanding before they are memorized. Good use of the blackboard and overhead projector is helpful during the teaching of this lesson.

Lessons 162 and 163 can be taught in three class periods if the lessons are integrated.

Lesson 164 — Similar Triangles

Note the comment on page 325 concerning the use of mathematical or physical measures. A review of the different ways that the vertices, angles, and sides of a triangle can be put in one-to-one correspondence is advisable before the material of this lesson is introduced.

The symbol " $\sim$ " is given a new meaning in this lesson. It is used here to mean, "is similar to". Previously it was given two other meanings, "is equivalent to" and "it is not the case that".

Be sure that the properties of similarity for triangles presented in the text are clearly understood. A property not considered in the lesson is described on page 199 of the *Teaching Guide.*

The *Teaching Guide* suggests that Exercises 10, 11, 13, and 16, page 330 be omitted.

This lesson can be taught in one class period, but the properties should be reviewed periodically.

Lessons 165-166 — Trigonometric Rate Pairs, Using a Table of Tangents, Sines, and Cosines

These two lessons are not included in the minimum program, but teachers who have taught the course suggest that they be taught if time permits, because they provide good review and practice in such areas as similar triangles and physical measures. The two lessons would likely require at least four teaching periods.

Lesson 167 — Problems Involving Indirect Measurement

Two class periods will be required for this lesson. Note that all but the first one and one-half pages of this lesson must be omitted if students have not studied trigonometric rate pairs. Students enjoy using their knowledge of similar triangles to solve problems involving indirect measurement.

Unit 13 - Formulas

(This whole unit is optional for an interim period.)

Unit 13 brings together a number of the mathematical ideas studied to date and focuses them on practical problem solving. The students develop, learn, and apply a number of formulas used in business, science and geometry.

The unit consists of ten lessons, three of which are omitted from the minimum program in their entirety (Lessons 169, 171, 172). Note that all Lessons 169 to 175 have some portions not included in the minimum program. If the class is on schedule at this time, the teacher may wish to teach some of the topics not included in the minimum program.

Lesson 168 — Relations Determined by $y = kx$

This lesson contains at least three concepts that are essential to the rest of the work of the unit. The teacher should be quite sure that the pupils grasp the meanings of the terms, direct variation, constant of variation, and locus of a solution set.

The *Teaching Guide* can be followed completely during the teaching of this lesson. Notice that Exercises 6 to 10 inclusive may be omitted except for good students. Since this lesson is basic to those that follow, it is suggested that two class periods be set aside for its development.

Lesson 169 — Relations Determined by $y = kx^2$

This lesson is not included in the minimum program. However, good classes enjoy the material included.

Lesson 170 — Relations Determined by $y = \frac{k}{x}$

In this lesson students study conditions for inverse variation. It is suggested that teachers prepare a list of practical situations in which one quantity decreases as the other quantity increases. Students can assist in this undertaking as an introduction to the lesson. Notice that conditions for inverse variation can be stated in the form $xy = k$, provided "k" cannot be replaced by zero. The conditions $xy = 0$ and $y = \frac{0}{x}$ do not have identical solution sets.

Read pages 209 to 211 of the *Teaching Guide* carefully. See the last paragraph, page 211, for parts that may be omitted from the minimum program.

Lessons 171-172 — Relations Determined by $y = \frac{k}{x^2}$, Relations Determined by Conditions for Variation

These two lessons are not included in the minimum program. They would supply excellent background for better students if time permitted their inclusion in the program.

Lesson 173 — Problems Involving Conditions for Variation

Note that the assumption is made that all measures given in the problems are mathematical measures, not physical measures. Problems involving conditions for variation of the types $y = kx^2$ and $y = \frac{k}{x^2}$ are omitted from the minimum program.

Lesson 174 — Formulas Derived from $\frac{y}{x} \sim \frac{k}{1}$

In this lesson students learn that a formula is a special kind of condition that may be used to solve many problems of the same kind. It is important that the formulas be developed using techniques similar to those outlined in the text, so that pupils form a bridge of understanding connecting their work with formulas with their previous mathematical learnings.

If the work with trigonometry has been omitted previously, the trigonometric formulas in this lesson can be omitted.

Lesson 175 — The Use of Formulas

In this lesson students apply the formulas that they developed in the last lesson to the solution of practical problems. Teachers may wish to develop solutions to problems in addition to those provided in the text, before assigning the exercises. Once again the problems involving trigonometry should be omitted if the students have not studied this topic earlier.

Lessons 176-177 — Business and Science Formulas, Perimeter and Area Formulas

The teacher should expect to spend some extra time on these lessons to ensure that they are covered in detail. All students leaving Grade VIII should have a clear understanding of how mathematics is applied to everyday problems. Many teachers supplement the material provided in the text. The teacher should be sure that the terminology of business and science used in the problems is clearly understoood before problems are assigned.

"Applying Mathematics", pages 391-392, should not be ignored. It will be necessary to examine the "checking up" exercises in detail in order that questions involving concepts not studied by the students can be eliminated from the tests before they are assigned.

Unit 14 - Probability and Statistics

Only two lessons of this unit are included in the minimum program. The first deals with statistical graphs (line, circle, bar). The second deals with the more common measures of central tendency. Lessons 178-182, inclusive, are not included in the minimum program.

Lesson 183 — Organization and Presentation of Data

In this lesson students study the types of graphs that they are likely to encounter in books and newspapers. It is important that pupils be able to read such graphs quickly with understanding. While most students will have had contact with line, circle, and bar graphs, it is quite likely that many will not have undertaken any formal study of them.

The presentation of this lesson may take three or more class periods depending on the amount of supplementary material supplied by the teacher. Students who are required to prepare neat graphs with appropriate scales and labels will likely develop the best understanding. The main purpose of the lesson presented in the text is to give students some understanding of the kind of graph that is appropriate for picturing a particular kind of data. There are many kinds of visual aids that can add interest to lessons in this section. A week or two prior to the teaching of the lesson, for example, pupils might prepare a display of graphs found in newspapers and magazines.

Lesson 184 — The Arithmetic Mean

In this lesson students are introduced in a small way to statistical analysis. They study two measures of central tendency, the general formula for the arithmetic mean, and the meaning of the range of a set of measures. The teacher should show that the use of the mean, or average, is more appropriate in some situations than others. If time permits, the idea of mode could be introduced as suggested on page 240 of the *Teaching Guide*. Many classes will be able to complete this lesson in two periods.

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